Logarithmen ohne Hilfsmittel

1a) $\log_2(\sqrt[3]{4}) = \log_2(4^{\frac{1}{3}}) \Leftrightarrow 2^x = 4^{\frac{1}{3}}$

$\Leftrightarrow 2^x = (2^2)^{\frac{1}{3}} = 2^{\frac{2}{3}}$

$\rightarrow \underline{\underline{x = \frac{2}{3}}}$

b) $\log_{\sqrt{2}}\left(\frac{1}{64}\right) \Leftrightarrow (\sqrt{2})^x = 2^{-6}$

$\Leftrightarrow 2^{\frac{x}{2}} = 2^{-6} \rightarrow \underline{\underline{x = -12}}$

c) $\log_4(8 \cdot \sqrt[4]{32}) = \log_4(2^3 \cdot 2^{\frac{5}{4}}) = \log_4(2^{\frac{17}{4}})$

$\rightarrow 4^x = 2^{\frac{17}{4}}$

$2^{2x} = 2^{\frac{17}{4}} \rightarrow \underline{\underline{x = \frac{17}{8}}} = 2{,}125$

d) $\log_{\frac{1}{4}}(16 \cdot \sqrt{2}) = \log_{\frac{1}{4}}(2^4 \cdot 2^{\frac{1}{2}}) = \log_{\frac{1}{4}}(2^{\frac{9}{2}})$

$\rightarrow \left(\frac{1}{4}\right)^x = 2^{\frac{9}{2}}$

$(2^{-2})^x = 2^{\frac{9}{2}} \rightarrow 2^{-2x} = 2^{\frac{9}{2}} \rightarrow \underline{\underline{x = -\frac{9}{4}}}$

e) $\log_{\sqrt{3}}(81^{\frac{1}{5}}) = \log_{\sqrt{3}}(3^{\frac{4}{5}})$

$\rightarrow (\sqrt{3})^x = 3^{\frac{4}{5}} \rightarrow (3^{\frac{1}{2}})^x = 3^{\frac{4}{5}} \rightarrow \underline{\underline{x = \frac{8}{5}}}$

f) $4^{\frac{\ln 27}{\ln 8}} = 4^{\frac{\ln 3^3}{\ln 2^3}}$ $\quad 4^{\frac{3 \cdot \ln 3}{3 \cdot \ln 2}} = 4^{\log_2(3)}$

$= 2^{2 \cdot \log_2(3)} = 2^{\log_2(3^2)} = \underline{\underline{3^2 = 9}}$

g) $\log_{\sqrt{5}}\left(e^{-2\cdot\ln(25)}\right) = \log_{\sqrt{5}}\left(e^{\ln\left(\left(\frac{1}{25}\right)^2\right)}\right) = \log_{\sqrt{5}}\left(\frac{1}{25}\right)^2$

$\rightarrow \sqrt{5}^{\,x} = \left(5^{-2}\right)^2 \rightarrow \underline{\underline{x = -4}}$

h) $\log_{\sqrt{2}}\left(e^{-3\cdot\ln(8)}\right) = \log_{\sqrt{2}}\left(e^{\ln(8^{-3})}\right)$

$= \log_{\sqrt{2}}\left(8^{-3}\right) \rightarrow \left(2^{\frac{1}{2}}\right)^x = 8^{-3}$

$\rightarrow 2^{\frac{x}{2}} = 2^{-9} \rightarrow \underline{\underline{x = -18}}$

2a) $\log_4(x-1) + \log_4(16) = \log_4(6x+1)$

$\log_4(16\cdot(x-1)) = \log_4(6x+1)$

$\rightarrow 16x - 16 = 6x + 1 \rightarrow \underline{\underline{x = 1{,}7}}$

b) $0 = \ln\frac{2x}{1-3x} \rightarrow 1 = \frac{2x}{1-3x} \rightarrow 1 - 3x = 2x \rightarrow \underline{\underline{x = \frac{1}{5}}}$

c) $x = \log_{\frac{1}{5}}\left(5^{\frac{2}{3}}\right) \rightarrow \left(5^{-1}\right)^x = 5^{\frac{2}{3}} \rightarrow -x = \frac{2}{3} \quad \underline{\underline{x = -\frac{2}{3}}}$

d) $\log(2x+4) - \log(x) = \log(x+5) - \log(x-1)$

$\rightarrow \log\left(\frac{2x+4}{x}\right) = \log\left(\frac{x+5}{x-1}\right)$

$\rightarrow (2x+4)(x-1) = x(x+5)$

$\rightarrow 2x^2 - 2x + 4x - 4 = x^2 + 5x$

$\rightarrow x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0 \quad \underline{\underline{L = \{4; \cancel{-1}\}}}$

Probe! x=-1 darf nicht in die Gleichung eingesetzt werden.

e) $\ln\left(\frac{(3x+3)(x+2)}{x^2-1}\right) = \ln(6) \rightarrow (3x+3)(x+2) = 6x^2 - 6$

$\rightarrow 3x^2 + 9x + 6 = 6x^2 - 6 \rightarrow 0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x = -1$ nicht erlaubt $\quad \underline{\underline{L = \{4\}}}$