

Logarithmen ohne Hilfsmittel

$$\begin{aligned} 1a) \log_2(\sqrt[3]{74}) &= \log_2(4^{\frac{1}{2}}) \iff 2^x = 4^{\frac{1}{2}} \\ &\iff 2^x = (2^2)^{\frac{1}{2}} = 2^{\frac{1}{2}} \\ &\rightarrow x = \underline{\underline{\frac{2}{3}}} \end{aligned}$$

$$\begin{aligned} b) \log_{\sqrt{2}}\left(\frac{1}{64}\right) &\iff (\sqrt{2})^x = 2^{-6} \\ &\iff 2^{\frac{x}{2}} = 2^{-6} \rightarrow x = \underline{\underline{-12}} \end{aligned}$$

$$\begin{aligned} c) \log_4(8 \cdot \sqrt[4]{32}) &= \log_4(2^3 \cdot 2^{\frac{5}{4}}) = \log_4(2^{\frac{17}{4}}) \\ &\rightarrow 4^x = 2^{\frac{17}{4}} \\ &2^{2x} = 2^{\frac{17}{4}} \rightarrow x = \underline{\underline{\frac{17}{8}}} = 2,125 \end{aligned}$$

$$\begin{aligned} d) \log_{\frac{1}{4}}(16 \cdot \sqrt{2}) &= \log_{\frac{1}{4}}(2^4 \cdot 2^{\frac{1}{2}}) = \log_{\frac{1}{4}}(2^{\frac{9}{2}}) \\ &\rightarrow \left(\frac{1}{4}\right)^x = 2^{\frac{9}{2}} \\ &(2^{-2})^x = 2^{\frac{9}{2}} \rightarrow 2^{-2x} = 2^{\frac{9}{2}} \rightarrow x = \underline{\underline{-\frac{9}{4}}} \end{aligned}$$

$$\begin{aligned} e) \log_{\sqrt{3}}(81^{\frac{1}{5}}) &= \log_{\sqrt{3}}(3^{\frac{4}{5}}) \\ &\rightarrow (\sqrt{3})^x = 3^{\frac{4}{5}} \rightarrow (3^{\frac{1}{2}})^x = 3^{\frac{4}{5}} \rightarrow x = \underline{\underline{\frac{8}{5}}} \end{aligned}$$

$$\begin{aligned} f) 4 \frac{\ln 27}{\ln 8} &= 4 \frac{\ln 3^3}{\ln 2^3} = 4 \frac{3 \cdot \ln 3}{3 \cdot \ln 2} = 4 \log_2(3) \\ &= 2 \cdot 2 \log_2(3) = 2 \log_2(3^2) = \underline{\underline{3^2 = 9}} \end{aligned}$$

$$g) \log_5 (e^{-2 \cdot \ln(25)}) = \log_5 (e^{\ln((\frac{1}{25})^2)}) = \log_5 (\frac{1}{25})^2$$

$$\rightarrow 5^x = (5^{-2})^2 \rightarrow \underline{\underline{x = -4}}$$

$$h) \log_{\sqrt{2}} (e^{-3 \cdot \ln(8)}) = \log_{\sqrt{2}} (e^{\ln(8^{-3})})$$

$$= \log_{\sqrt{2}} (8^{-3}) \rightarrow (2^{\frac{1}{2}})^x = 8^{-3}$$

$$\rightarrow 2^{\frac{x}{2}} = 2^{-9} \rightarrow \underline{\underline{x = -18}}$$

$$2a) \log_4 (x-1) + \log_4 (16) = \log_4 (6x+1)$$

$$\log_4 (16 \cdot (x-1)) = \log_4 (6x+1)$$

$$\rightarrow 16x - 16 = 6x + 1 \rightarrow \underline{\underline{x = 1,7}}$$

$$b) 0 = \ln \frac{2x}{1-3x} \rightarrow 1 = \frac{2x}{1-3x} \rightarrow 1-3x = 2x \rightarrow x = \underline{\underline{\frac{1}{5}}}$$

$$c) x = \log_5 (5^{\frac{2}{5}}) \rightarrow (5^{-1})^x = 5^{\frac{2}{5}} \rightarrow -x = \frac{2}{5} \quad \underline{\underline{x = -\frac{2}{5}}}$$

$$d) \log(2x+4) - \log(x) = \log(x+5) - \log(x-1)$$

$$\rightarrow \log\left(\frac{2x+4}{x}\right) = \log\left(\frac{x+5}{x-1}\right)$$

$$\rightarrow (2x+4)(x-1) = x(x+5)$$

$$\rightarrow 2x^2 - 2x + 4x - 4 = x^2 + 5x$$

$$\rightarrow x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0 \quad \underline{\underline{L = \{4, -1\}}}$$

$$e) \ln\left(\frac{(3x+3)(x+2)}{x^2-1}\right) = \ln(6) \rightarrow (3x+3)(x+2) = 6x^2-6$$

$$\rightarrow 3x^2 + 9x + 6 = 6x^2 - 6 \rightarrow 0 = x^2 - 3x - 4$$

$$0 = (x-4)(x+1)$$

$x = -1$ nicht erlaubt $\underline{\underline{L = \{4\}}}$