Lösung zu Aufgabe 420 ex-quotientenregel-anwenden

a) $f'(x) = \left(\frac{\ln(x)}{x^3}\right)' = \frac{\frac{1}{x}x^3 - \ln(x)\cdot 3x^2}{(x^3)^2} = \frac{x^2 - 3x^2\cdot\ln(x)}{x^6} = \frac{x^2(1-3\ln(x))}{x^6} = \frac{1-3\ln(x)}{x^4}$

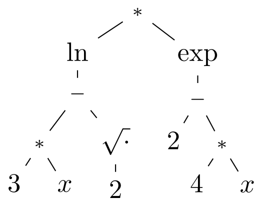
b) $f'(x) = \left(\frac{x^5}{x^3}\right)' = \frac{5x^4\cdot x^3 - x^5\cdot 3x^2}{(x^3)^2} = \frac{2x^7}{x^6} = 2x$ Zuerst vereinfachen wäre einfacher gewesen!

c) $f'(x) = \left(\frac{\log_2(x)}{2^x}\right)' = \frac{\frac{1}{\ln(2)x}\cdot 2^x - \log_2(x)\cdot\ln(2)\cdot 2^x}{(2^x)^2} = \frac{2^x\left(\frac{1}{\ln(2)x} - \frac{\ln(x)}{\ln(2)}\cdot\ln(2)\right)}{(2^x)^2} = \frac{\frac{1}{\ln(2)x} - \ln(x)}{2^x}$

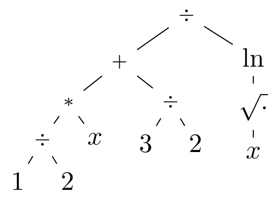
d) $f'(x) = \left(\frac{\mathrm{e}^x}{x^2}\right)' = \frac{\mathrm{e}^x\cdot x^2 - \mathrm{e}^x\cdot 2x}{(x^2)^2} = \frac{x\cdot\mathrm{e}^x(x-2)}{x^4} = \mathrm{e}^x\cdot\frac{x-2}{x^3}$

Lösung zu Aufgabe 421 ex-termbaueme-zeichnen

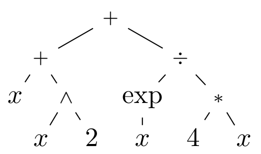
a) $\ln\left(3\cdot x - \sqrt{2}\right)\cdot\mathrm{e}^{2-4\cdot x}$



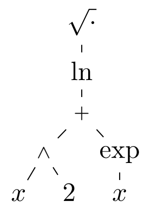
b) $\frac{\frac{1}{2}\cdot x + \frac{3}{2}}{\ln\left(\sqrt{x}\right)}$



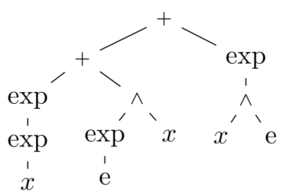
c) $x + x^2 + \frac{\mathrm{e}^x}{4\cdot x}$



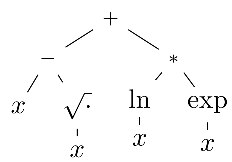
d) $\sqrt{\ln\left(x^2 + \mathrm{e}^x\right)}$



e) $\mathrm{e}^{\mathrm{e}^x} + (\mathrm{e}^{\mathrm{e}})^x + \mathrm{e}^{x^{\mathrm{e}}}$



f) $x - \sqrt{x} + \ln(x)\cdot\mathrm{e}^x$



Lösung zu Aufgabe 422 ex-dem-teufel-ein-ohr-ableiten

a) Kettenregel. Äussere Funktion: e^x, innere Funktion $2x$. Also $\left(\mathrm{e}^{2x}\right)' = \mathrm{e}^{2x}\cdot 2$

b) Produktregel. $(x\cdot 2^x)' = 1\cdot 2^x + x\cdot\ln(2)\cdot 2^x = 2^x(1+\ln(2)x)$.

c) Quotientenregel. $\left(\frac{\ln(x)}{x}\right)' = \frac{\frac{1}{x}\cdot x - \ln(x)\cdot 1}{x^2} = \frac{1-\ln(x)}{x^2}$

d) $\left(\frac{\sqrt{\mathrm{e}^x}}{\ln(4x)}\right)' = \frac{\frac{1}{2\sqrt{\mathrm{e}^x}}\cdot\mathrm{e}^x\cdot\ln(4x) - \sqrt{\mathrm{e}^x}\cdot\frac{1}{4x}\cdot 4}{(\ln(4x))^2} = \frac{\frac{1}{2}\cdot\sqrt{\mathrm{e}^x}\ln(4x) - \sqrt{\mathrm{e}^x}\cdot\frac{1}{x}}{(\ln(4x))^2}$

e) $\left(\ln\left(\sqrt{2^x}\right)\right)' = \frac{1}{\sqrt{2^x}}\cdot\frac{1}{2\sqrt{2^x}}\cdot\ln(2)\cdot 2^x = \frac{\ln(2)\cdot 2^x}{2\cdot 2^x} = \frac{\ln(2)}{2}$

Das könnte man auch billiger haben, indem man zuerst vereinfacht: $\ln\left(\sqrt{2^x}\right) = \ln\left(2^{\frac{x}{2}}\right) = \frac{x}{2}\cdot\ln(2) = x\cdot\frac{2}{\ln(2)}$.

f) $\left(\mathrm{e}^{x^2}\cdot\sqrt{x+\frac{1}{x}}\right)' = \mathrm{e}^{x^2}\cdot 2x\cdot\sqrt{x+\frac{1}{x}} + \mathrm{e}^{x^2}\cdot\frac{1}{2\sqrt{x+\frac{1}{x}}}\cdot\left(1-\frac{1}{x^2}\right)$