e) $\dfrac{b-c}{a^2+ac} - \dfrac{a-b}{ac+c^2} + \dfrac{a^2+c^2}{a^2c+ac^2} = \dfrac{b-c}{a(a+c)} - \dfrac{a-b}{c(a+c)} + \dfrac{a^2+c^2}{ac(a+c)} = \dfrac{c(b-c)-a(a-b)+(a^2+c^2)}{ac(a+c)} = \dfrac{cb-c^2-(a^2-ab)+a^2+c^2}{ac(a+c)} = \dfrac{cb-a^2+ab+a^2}{ac(a+c)} = \dfrac{cb+ab}{ac(a+c)} = \dfrac{b(c+a)}{ac(a+c)} = \dfrac{b}{ac}$

f) $\dfrac{3s}{(s-2)^2} - \dfrac{2}{s} + \dfrac{s+4}{2s-s^2} = \dfrac{3s}{(s-2)^2} - \dfrac{2}{s} + \dfrac{s+4}{s(2-s)} = \dfrac{3s}{(s-2)^2} - \dfrac{2}{s} - \dfrac{s+4}{s(s-2)} = \dfrac{3s^2-2(s-2)^2-(s+4)(s-2)}{s(s-2)^2} = \dfrac{3s^2-2(4-4s+s^2)-(s^2+2s-8)}{s(s-2)^2} = \dfrac{3s^2-(8-8s+2s^2)-s^2-2s+8}{s(s-2)^2} = \dfrac{3s^2-8+8s-2s^2-s^2-2s+8}{s(s-2)^2} = \dfrac{6s}{s(s-2)^2} = \dfrac{6}{(s-2)^2}$

Hinweis: Anstatt Vorzeichenakrobatik $(2-s) = -(2-s)$, könnte man auch einfach $(2-s)^2 = (-(s-2))^2 = (s-2)^2$ verwenden.

Lösung zu Aufgabe 7.9 ex-bruchterme-querbeet

a) $\dfrac{u^2-v^2}{u^2+v^2}\left(\dfrac{u}{u+v}+\dfrac{v}{u-v}\right) = \dfrac{(u+v)(u-v)}{u^2+v^2}\left(\dfrac{u(u-v)}{(u+v)(u-v)}+\dfrac{v(u+v)}{(u+v)(u-v)}\right) = \dfrac{(u+v)(u-v)}{u^2+v^2}\left(\dfrac{u^2-uv+uv+v^2}{(u+v)(u-v)}\right) = \dfrac{(u+v)(u-v)}{u^2+v^2}\cdot\dfrac{u^2+v^2}{(u+v)(u-v)} = 1$

b) $\left(\dfrac{1}{r-s}-\dfrac{1}{r+s}\right)^2 = \left(\dfrac{r+s}{(r-s)(r+s)}-\dfrac{r-s}{(r+s)(r-s)}\right)^2 = \left(\dfrac{r+s-(r-s)}{(r-s)(r+s)}\right)^2 = \left(\dfrac{2s}{(r-s)(r+s)}\right)^2 = \dfrac{4s^2}{(r-s)^2(r+s)^2}$

c) $\left(\dfrac{a}{b}-\dfrac{c}{d}\right):\left(\dfrac{a}{b}+\dfrac{c}{d}\right) = \dfrac{ad-bc}{bd}:\dfrac{ad+cb}{bd} = \dfrac{ad-bc}{bd}\cdot\dfrac{bd}{ad+cb} = \dfrac{ad-bc}{ad+cb}$

d) $\left(1-\dfrac{1}{n^2}\right):\left(1+\dfrac{1}{n}\right) = \dfrac{n^2-1}{n^2}:\dfrac{n+1}{n} = \dfrac{(n+1)(n-1)}{n^2}\cdot\dfrac{n}{n+1} = \dfrac{n-1}{n}$

e) $\left(a^2+2+\dfrac{1}{a^2}\right):\left(a^2-\dfrac{1}{a^2}\right) = \left(a+\dfrac{1}{a}\right)^2:\left(a+\dfrac{1}{a}\right)\left(a-\dfrac{1}{a}\right) = \left(a+\dfrac{1}{a}\right):\left(a-\dfrac{1}{a}\right) = \dfrac{a^2+1}{a}:\dfrac{a^2-1}{a} = \dfrac{a^2+1}{a}\cdot\dfrac{a}{a^2-1} = \dfrac{a^2+1}{(a+1)(a-1)}$

f) $\dfrac{1}{n^2+2n+1}\cdot\left(\dfrac{n(n+1)}{2}+\dfrac{(n+1)(n+2)}{2}\right) = \dfrac{1}{(n+1)^2}\cdot\left(\dfrac{n^2+n+n^2+3n+2}{2}\right) = \dfrac{1}{(n+1)^2}\cdot\left(\dfrac{2n^2+4n+2}{2}\right) = \dfrac{1}{(n+1)^2}\cdot\left(\dfrac{2(n^2+2n+1)}{2}\right) = \dfrac{1}{(n+1)^2}\cdot(n+1)^2 = 1$

g) $\left(\dfrac{3n^2(n+1)-2n(n^2+4)}{n+1}+12\right):(n^2+4)-\dfrac{2}{n+1}+\dfrac{1-n^2}{n^2+n} = \left(3n^2-\dfrac{2n(n^2+4)}{n+1}+12\right):(n^2+4)-\dfrac{2}{n+1}+\dfrac{(1+n)(1-n)}{n(n+1)} = \dfrac{3n^2}{n^2+4}-\dfrac{2n}{n+1}+\dfrac{12}{n^2+4}-\dfrac{2}{n+1}+\dfrac{1-n}{n} = \dfrac{3n^2+12}{n^2+4}-\dfrac{2n+2}{n+1}+\dfrac{1-n}{n} = \dfrac{3(n^2+4)}{n^2+4}-\dfrac{2(n+1)}{n+1}+\dfrac{1-n}{n} = 3-2+\dfrac{1-n}{n} = 1+\dfrac{1-n}{n} = \dfrac{n}{n}+\dfrac{1-n}{n} = \dfrac{n+1-n}{n} = \dfrac{1}{n}$