



$$\begin{aligned} \text{e) } \frac{b-c}{a^2+ac} - \frac{a-b}{ac+c^2} + \frac{a^2+c^2}{a^2c+ac^2} &= \frac{b-c}{a(a+c)} - \frac{a-b}{c(a+c)} + \frac{a^2+c^2}{ac(a+c)} = \\ \frac{c(b-c) - a(a-b) + (a^2+c^2)}{ac(a+c)} &= \frac{cb - c^2 - (a^2 - ab) + a^2 + c^2}{ac(a+c)} = \frac{cb - a^2 + ab + a^2}{ac(a+c)} = \frac{cb + ab}{ac(a+c)} = \\ \frac{b(c+a)}{ac(a+c)} &= \frac{b}{ac} \end{aligned}$$

$$\begin{aligned} \text{f) } \frac{3s}{(s-2)^2} - \frac{2}{s} + \frac{s+4}{2s-s^2} &= \frac{3s}{(s-2)^2} - \frac{2}{s} + \frac{s+4}{s(2-s)} = \frac{3s}{(s-2)^2} - \frac{2}{s} - \frac{s+4}{s(s-2)} = \\ \frac{3s^2 - 2(s-2)^2 - (s+4)(s-2)}{s(s-2)^2} &= \frac{3s^2 - 2(4 - 4s + s^2) - (s^2 + 2s - 8)}{s(s-2)^2} = \\ \frac{3s^2 - (8 - 8s + 2s^2) - s^2 - 2s + 8}{s(s-2)^2} &= \frac{3s^2 - 8 + 8s - 2s^2 - s^2 - 2s + 8}{s(s-2)^2} = \frac{6s}{s(s-2)^2} = \frac{6}{(s-2)^2} \end{aligned}$$

Hinweis: Anstatt Vorzeichenakrobatik $(2-s) = -(2-s)$, könnte man auch einfach $(2-s)^2 = (-(s-2))^2 = (s-2)^2$ verwenden.

✂ Lösung zu Aufgabe 7.9 ex-bruchterme-querbeet

$$\begin{aligned} \text{a) } \frac{u^2 - v^2}{u^2 + v^2} \left(\frac{u}{u+v} + \frac{v}{u-v} \right) &= \frac{(u+v)(u-v)}{u^2 + v^2} \left(\frac{u(u-v)}{(u+v)(u-v)} + \frac{v(u+v)}{(u+v)(u-v)} \right) = \\ \frac{(u+v)(u-v)}{u^2 + v^2} \left(\frac{u^2 - uv + uv + v^2}{(u+v)(u-v)} \right) &= \frac{(u+v)(u-v)}{u^2 + v^2} \cdot \frac{u^2 + v^2}{(u+v)(u-v)} = 1 \end{aligned}$$

$$\begin{aligned} \text{b) } \left(\frac{1}{r-s} - \frac{1}{r+s} \right)^2 &= \left(\frac{r+s}{(r-s)(r+s)} - \frac{r-s}{(r+s)(r-s)} \right)^2 = \left(\frac{r+s - (r-s)}{(r-s)(r+s)} \right)^2 = \\ \left(\frac{2s}{(r-s)(r+s)} \right)^2 &= \frac{4s^2}{(r-s)^2(r+s)^2} \end{aligned}$$

$$\text{c) } \left(\frac{a}{b} - \frac{c}{d} \right) : \left(\frac{a}{b} + \frac{c}{d} \right) = \frac{ad - bc}{bd} : \frac{ad + cb}{bd} = \frac{ad - bc}{bd} \cdot \frac{bd}{ad + cb} = \frac{ad - bc}{ad + cb}$$

$$\text{d) } \left(1 - \frac{1}{n^2} \right) : \left(1 + \frac{1}{n} \right) = \frac{n^2 - 1}{n^2} : \frac{n+1}{n} = \frac{(n+1)(n-1)}{n^2} \cdot \frac{n}{n+1} = \frac{n-1}{n}$$

$$\begin{aligned} \text{e) } \left(a^2 + 2 + \frac{1}{a^2} \right) : \left(a^2 - \frac{1}{a^2} \right) &= \left(a + \frac{1}{a} \right)^2 : \left(a + \frac{1}{a} \right) \left(a - \frac{1}{a} \right) = \left(a + \frac{1}{a} \right) : \left(a - \frac{1}{a} \right) = \\ \frac{a^2 + 1}{a} : \frac{a^2 - 1}{a} &= \frac{a^2 + 1}{a} \cdot \frac{a}{a^2 - 1} = \frac{a^2 + 1}{(a+1)(a-1)} \end{aligned}$$

$$\begin{aligned} \text{f) } \frac{1}{n^2 + 2n + 1} \cdot \left(\frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} \right) &= \frac{1}{(n+1)^2} \cdot \left(\frac{n^2 + n + n^2 + 3n + 2}{2} \right) = \\ \frac{1}{(n+1)^2} \cdot \left(\frac{2n^2 + 4n + 2}{2} \right) &= \frac{1}{(n+1)^2} \cdot \left(\frac{2(n^2 + 2n + 1)}{2} \right) = \frac{1}{(n+1)^2} \cdot (n+1)^2 = 1 \end{aligned}$$

$$\begin{aligned} \text{g) } \left(\frac{3n^2(n+1) - 2n(n^2+4)}{n+1} + 12 \right) : (n^2+4) - \frac{2}{n+1} + \frac{1-n^2}{n^2+n} &= \\ \left(3n^2 - \frac{2n(n^2+4)}{n+1} + 12 \right) : (n^2+4) - \frac{2}{n+1} + \frac{(1+n)(1-n)}{n(n+1)} &= \\ \frac{3n^2}{n^2+4} - \frac{2n}{n+1} + \frac{12}{n^2+4} - \frac{2}{n+1} + \frac{1-n}{n} &= \frac{3n^2+12}{n^2+4} - \frac{2n+2}{n+1} + \frac{1-n}{n} = \\ \frac{3(n^2+4)}{n^2+4} - \frac{2(n+1)}{n+1} + \frac{1-n}{n} &= 3 - 2 + \frac{1-n}{n} = 1 + \frac{1-n}{n} = \frac{n}{n} + \frac{1-n}{n} = \frac{n+1-n}{n} = \frac{1}{n} \end{aligned}$$