a) $\log_2(32) = \log_2(2^5) = 5$

b) $\log_3\left(\frac{1}{81}\right) = \log_3\left(\frac{1}{3^4}\right) = \log_3\left(3^{-4}\right) = -4$

c) $\log_5(\sqrt{5}) = \log_5\left(5^{\frac{1}{2}}\right) = \frac{1}{2}$

d) $\log_9(27) = \log_9\left(3^3\right) = \log_9\left(\left(\left(3^2\right)^{\frac{1}{2}}\right)^3\right) = \log_9\left(9^{\frac{3}{2}}\right) = \frac{3}{2}$

e) $\log_2\left(\frac{1}{\sqrt[3]{16}}\right) = \log_2\left(\frac{1}{\sqrt[3]{2^4}}\right) = \log_2\left(\frac{1}{2^{\frac{4}{3}}}\right) = \log_2\left(2^{-\frac{4}{3}}\right) = -\frac{4}{3}$

f) $\log_7(1) = \log_7\left(7^0\right) = 0$

Lösung zu Aufgabe 18.17 ex-logarithmen-von-hand2

a) $3^{\log_3(7)} = 7$

b) $9^{\log_3(\sqrt{5})} = \left(3^2\right)^{\log_3(\sqrt{5})} = 3^{2\cdot\log_3(\sqrt{5})} = \left(3^{\log_3(\sqrt{5})}\right)^2 = \left(\sqrt{5}\right)^2 = 5$

c) $2^{-\log_8(125)} = \left(8^{\frac{1}{3}}\right)^{-\log_8(125)} = \left(8^{-\log_8(125)}\right)^{\frac{1}{3}} = \left(\frac{1}{8^{\log_8(125)}}\right)^{\frac{1}{3}} = \frac{1}{\sqrt[3]{125}} = \frac{1}{5}$

Lösung zu Aufgabe 18.18 ex-logarithmen-zeichnen

Lösung zu Aufgabe 18.25 ex-logxy-und-logyx

Basiswechsel zur Basis e:
$\log_x(y) = \frac{\ln(y)}{\ln(x)}$ und $\log_y(x) = \frac{\ln(x)}{\ln(y)}$. Damit gilt

$$\log_x(y) = \frac{1}{\log_y(x)}$$

Oder mit der **Definition des Logarithmus**:
Sei $a = \log_x(y)$ und $b = \log_y(x)$. Damit gilt $x^a = y$ und $y^b = x$. Potenziert man die erste Gleichung mit $\frac{1}{a}$ erhält man $x = y^{\frac{1}{a}}$ und damit $y^b = y^{\frac{1}{a}}$, also $b = \frac{1}{a}$ und damit $\log_x(y) = \frac{1}{\log_y(x)}$.

Lösung zu Aufgabe 18.26 ex-logarithmen-zerlegen

a) $\log\left(\frac{ab}{a+b}\right) = \log(a) + \log(b) - \log(a+b)$

b) $\log\left(\frac{\sqrt[4]{x}}{x^2-y^2}\right) = \frac{1}{4}\log(x) - \log(x+y) - \log(x-y)$

c) $\log_a\left(\frac{a\cdot b^c}{\sqrt[n]{a}}\right) = 1 + c\log_a(b) - \frac{1}{n} = \frac{n-1}{n} + c\log_a(b)$

d) $\ln\left(\frac{1}{2}\right) + \ln\left(\frac{2}{3}\right) + \ln\left(\frac{3}{4}\right) + \ldots + \ln\left(\frac{999}{1000}\right) = (\ln(1) - \ln(2)) + (\ln(2) - \ln(3)) + \ldots + \ln(999) - \ln(1000) = \ln(1) - \ln(1000) = 0 - \ln\left((2\cdot 5)^3\right) - 3\cdot(\ln(2) + \ln(5))$

Lösung zu Aufgabe 18.27 ex-logarithmen-zusammenfassen

a) $\log(b) - \log(c+d) = \log\left(\frac{b}{c+d}\right)$

b) $2\log(x) + 3\log(y) - 5\log(z) = \log\left(x^2\right) + \log\left(y^3\right) - \log\left(z^5\right) = \log\left(\frac{x^2y^3}{z^5}\right)$

c) $\frac{1}{3}(\log(b) + 2\log(c)) - \frac{1}{2}(5\log(d) + \log(f)) = \log\left(\frac{\sqrt[3]{b\cdot c^2}}{\sqrt{d^5 f}}\right)$

d) $\ln(a+b) + 1 = \ln(a+b) + \ln(\mathrm{e}) = \ln(\mathrm{e}(a+b))$