



**\* Lösung zu Aufgabe 18.30** ex-log-und-exp-gleichungen

a)

$$\begin{aligned}
 2^x &= e^7 && | \ln(\cdot) \\
 x \cdot \ln(2) &= 7 && | : \ln(2) \\
 x &= \frac{7}{\ln(2)} \approx 10.10
 \end{aligned}$$

b)

$$\begin{aligned}
 3^{x+3} &= 2^{2x} && | \ln(\cdot) \\
 (x+3) \ln(3) &= 2x \ln(2) && | - 2 \ln(2)x \\
 x(\ln(3) - 2 \ln(2)) + 3 \ln(3) &= 0 && | - 3 \ln(3) \\
 x(\ln(3) - 2 \ln(2)) &= -3 \ln(3) && | : (\ln(3) - 2 \ln(2)) \\
 x &= -\frac{\ln(3^3)}{\ln\left(\frac{3}{2^2}\right)} = \frac{\ln(27)}{\ln\left(\frac{3}{4}\right)}
 \end{aligned}$$

c)

$$\begin{aligned}
 3^{x+2} \cdot 2^{x+3} &= 4^{x+4} && | \ln(\cdot) \\
 \ln(3^{x+2}) + \ln(2^{x+3}) &= (x+4) \ln(4) \\
 (x+2) \ln(3) + (x+3) \ln(2) &= (x+4) \ln(4) \\
 x(\ln(3) + \ln(2)) + 2 \ln(3) + 3 \ln(2) &= x \ln(4) + 4 \ln(4) && | - x \ln(4) - 2 \ln(3) - 3 \ln(2) \\
 x \ln(1.5) &= 4 \ln(4) - 2 \ln(3) - 3 \ln(2) && | : \ln(1.5) \\
 x &= \frac{\ln\left(\frac{4^4}{3^2 \cdot 2^3}\right)}{\ln(1.5)} \\
 x &= \log_{1.5}\left(\frac{2^5}{3^2}\right) \approx 3.1285
 \end{aligned}$$

d)

$$\begin{aligned}
 (\sqrt{2^{x-1}})^{x+1} &= \sqrt{3^x} \\
 \sqrt{2^{x^2-1}} &= \sqrt{3^x} && | \ln(\cdot) \\
 (x^2-1) \cdot \frac{1}{2} \ln(2) &= x \cdot \frac{1}{2} \cdot \ln(3) && | - x \cdot \frac{1}{2} \cdot \ln(3) \\
 x^2 \cdot \frac{1}{2} \ln(2) - x \cdot \frac{1}{2} \cdot \ln(3) - \frac{1}{2} \ln(2) &= 0 && | \cdot 2 \\
 \ln(2)x^2 - \ln(3)x - \ln(2) &= 0 && | \text{quadratische Gleichung} \\
 x_{1,2} &= \frac{\ln(3) \pm \sqrt{(\ln(3))^2 + 4 \ln(2)^2}}{2 \ln(2)} \\
 x_1 &\approx 2.068 \\
 x_2 &\approx -0.4835
 \end{aligned}$$