Integralrechnung ohne Hilfsmittel

1) Siehe Theorieheft

2) $\int_0^{\frac{\pi}{2}} 2\cdot\cos(x)\,dx = 2\cdot\left[\sin x\right]_0^{\frac{\pi}{2}} = 2\cdot(1-0) = \underline{\underline{2}}$

$\int_0^1 (4x^4 - x + 1)\,dx = \left[4\cdot\frac{x^5}{5} - \frac{x^2}{2} + x\right]_0^1 = \frac{4}{5} - \frac{1}{2} + 1 = \underline{\underline{\frac{4}{5} + \frac{1}{2}}}$

$\int_0^2 (cx^3 + x - 1)\,dx = \left[c\cdot\frac{x^4}{4} + \frac{x^2}{2} - x\right]_0^2 = \underline{\underline{4c}}$

$\int_0^1 (ax^3 - 6x)\,dx = \left[a\,\frac{x^4}{4} - 6\,\frac{x^2}{2}\right]_0^1 = \underline{\underline{\frac{a}{4} - \frac{6}{2}}}$

$\int_{-2}^{-1} \frac{9-3x^2}{x^4}\,dx = \int_{-2}^{-1} (9x^{-4} - 3x^{-2})\,dx = \left[-3x^{-3} + 3x^{-1}\right]_{-2}^{-1} = \underline{\underline{\frac{9}{8}}}$

$\int_{-4}^{-1} \frac{16-4x^2}{x^4}\,dx = \int_{-4}^{-1} (16x^{-4} - 4x^{-2})\,dx = \left[\frac{16}{-3}\,x^{-3} - 4\,\frac{1}{-1}\,x^{-1}\right] = \underline{\underline{\frac{9}{4}}}$

$\int_0^1 x^{\frac{1}{3}}(x^2 - 4x)\,dx = \int_0^1 (x^{\frac{7}{3}} - 4\cdot x^{\frac{4}{3}})\,dx = \left[\frac{3}{10}\,x^{\frac{10}{3}} - \frac{12}{7}\,x^{\frac{7}{3}}\right] = \underline{\underline{-\frac{99}{70}}}$

$\int_{-3}^0 \sqrt{1-x}\,dx = \int_{-3}^0 (1-x)^{\frac{1}{2}}\,dx$ lineare Substitutionsregel

$= \left[(-1)\cdot\frac{1}{\frac{3}{2}}\,(1-x)^{\frac{3}{2}}\right]_{-3}^0 = -\frac{2}{3}\cdot 1^{\frac{3}{2}} + \frac{2}{3}\cdot 4^{\frac{3}{2}} = \underline{\underline{\frac{14}{3}}}$

3) $\int a\cdot\cos x\,dx = \underline{\underline{a\cdot\sin x}} + C$

$\int \frac{1}{x^3}(x-1)\,dx = \int\left(\frac{1}{x^2} - \frac{1}{x^3}\right)dx = \underline{\underline{-\frac{1}{x} + \frac{1}{2x^2}}} + C$

$\int \sin(at-6)\,dt = \underline{\underline{-\frac{1}{a}\cdot\cos(at-6)}} + C$

lin. Substitutionsregel

4) $f(x) = x^2 - 1$

$g(x) = x + 1$ (Steigung $m = 1$, y-Achsenab. $= 1$)

$$\int_{-1}^{2} (f(x) - g(x))\,dx = \int_{-1}^{2} (x^2 - x - 2)\,dx = -\frac{9}{2} \rightarrow \underline{\underline{A = \frac{9}{2}}}$$

5) a) Schnittpunkte $1 - x^2 = 3x^2 - 3$

$\rightarrow 4x^2 - 4 = 0$

$\rightarrow 4(x-1)(x+1) = 0 \rightarrow x_1 = 1 \quad x_2 = -1$

$$\int_{-1}^{1} (4x^2 - 4)\,dx = \left[\frac{4}{3}x^3 - 4x\right]_{-1}^{1} = \left(\frac{4}{3} - 4\right) - \left(-\frac{4}{3} + 4\right) = -\frac{16}{3}$$

$\rightarrow \underline{\underline{A = \frac{16}{3}}}$

b) Schnittpunkte: $x^2 + 3x + \cancel{7} = 3x^2 + x + \cancel{7}$

$\rightarrow x(x-2)(x-1) = 0 \rightarrow x_1 = 0 \quad x_2 = 2 \quad x_3 = 1$

$$\int_{0}^{1} (x^3 - 3x^2 + 2x)\,dx = \left[\frac{x^4}{4} - \cancel{3}^{1} \cdot \frac{x^3}{\cancel{3}_{1}} + x^2\right]_{0}^{1} = \frac{1}{4}$$

$$\int_{1}^{2} (x^3 - 3x^2 + 2x)\,dx = \left[\frac{x^4}{4} - x^3 + x^2\right]_{1}^{2} = -\frac{1}{4}$$

$\rightarrow \underline{\underline{A = \frac{1}{2}}}$

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6) Nullstellen: $\frac{1}{4} x^3 - x = 0$

$$x\left(\frac{1}{4} x^2 - 1\right) = x\left(\frac{1}{2} x - 1\right)\left(\frac{1}{2} x + 1\right) = 0$$

$$\to \underline{\underline{x_1 = 0}} \quad \underline{\underline{x_2 = 2}} \quad \underline{\underline{x_3 = -2}}$$

$$f'(x) \overset{!}{=} 0 \to \frac{1}{4} 3x^2 - 1 = 0$$

$$\to x^2 = \frac{4}{3} \to \underline{\underline{x = \pm\sqrt{\frac{4}{3}}}}$$

$$\left|\int_{-1}^{0} f(x)\,dx\right| + \left|\int_{0}^{2} f(x)\,dx\right| = \left|\left[\frac{x^4}{16} - \frac{x^2}{2}\right]_{-1}^{0}\right| + \left|\left[\frac{x^4}{16} - \frac{x^2}{2}\right]_{0}^{2}\right|$$

$$= \left|-\frac{1}{16} + \frac{1}{2}\right| + |1 - 2| = \underline{\underline{\frac{23}{16}}}$$

7) Schnittpunkte $\frac{x^2}{\sqrt{a}} = a \cdot \sqrt{x}$

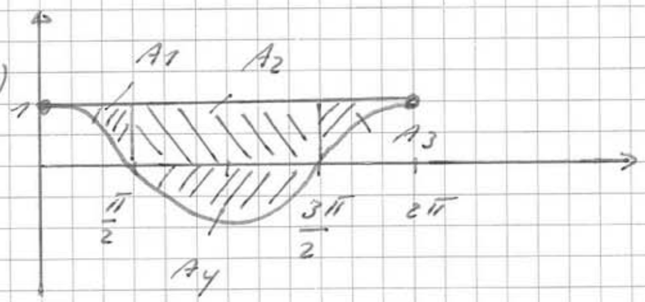
$$\to \frac{1}{\sqrt{a}} x^2 - a \cdot \sqrt{x} = 0$$

$$\to \sqrt{x}\left(\frac{1}{\sqrt{a}} x^{\frac{3}{2}} - a\right) = 0 \to \underline{x_1 = 0} \quad \underline{x_2 = a}$$

$$\left|\int_{0}^{a}\left(\frac{x^2}{\sqrt{a}} - a \cdot \sqrt{x}\right) dx\right| = \left|a^{\frac{5}{2}} \cdot \left(\frac{1}{3} - \frac{2}{3}\right)\right| \overset{!}{=} 1$$

$$\to a^{\frac{5}{2}} = 3 \to \underline{\underline{a = 3^{\frac{2}{5}}}}$$

8)



$$A_1 = \frac{\pi}{2} \cdot 1 - \int_{0}^{\frac{\pi}{2}} \cos x\,dx = \frac{\pi}{2} - [\sin x]_{0}^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$$

$$A_2 = \pi \cdot 1 \qquad A_3 = A_1 \qquad A_4 = \left|\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x\,dx\right| = 2$$

$$A_{tot} = A_1 + A_2 + A_3 + A_4 = \frac{\pi}{2} - 1 + \pi + \frac{\pi}{2} - 1 + 2 = \underline{\underline{2\pi}}$$

9) $\int_0^a \left(ax^3 - \frac{3}{2}x^2 - ax\right)dx = \left[a\frac{x^4}{4} - \frac{3}{2}\frac{x^3}{3} - a\frac{x^2}{2}\right]_0^a$

$= \frac{a^5}{4} - a^3 \overset{!}{=} 0 \longrightarrow a^3\left(\frac{a}{2}-1\right)\left(\frac{a}{2}+1\right) = 0$

$\longrightarrow a_1 = 0 \qquad \underline{\underline{a_2 = 2}} \qquad a_3 = -2$ (a_1 und a_3 sind nicht grösser null)

10) $F(a) = \int_{-a}^{a} (x^2+x+1)\,dx = \left[\frac{1}{3}x^3 - \frac{x^2}{2} + x\right]_{-a}^{a} = \underline{\frac{2}{3}a^3 + 2a}$

Extremalstellen: $F'(a) = 2a^2 + 2 = 0$ <u>unmöglich</u>

Wendestellen $F''(a) = 4a = 0 \longrightarrow \underline{\underline{a = 0}}$

$F'''(a) = 4$

11a) $f(x) = ax^3 + bx^2 + cx$ (y-Achsenabschn. $= 0$)

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

Maximum bei (3/0): $27a + 6b + c = 0$

Wendepunkt bei (2/-1): $12a + 2b = 0$

(3/0) liegt auf Kurve: $27a + 9b + 3c = 0$

$\longrightarrow a = -\frac{1}{2} \qquad b = 3 \qquad c = -\frac{9}{2} \qquad \left|\int_0^3 \left(-\frac{1}{2}x^3 + 3x^2 - \frac{9}{2}x\right)dx\right| = \underline{\underline{3,375}}$

b) $f(x) = ax^3 + bx^2 + cx$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

Maximum bei $x = 2$: $12a + 4b + c = 0$

P(1/-2) liegt auf Kurve: $a + b + c = -2$

(2/0) " " $8a + 4b + 2c = 0$

$\longrightarrow a = -2 \qquad b = 8 \qquad c = -8 \qquad \left|\int_0^2 (-2x^3 + 8x^2 - 8x)\,dx\right| = \underline{\underline{\frac{8}{3}}}$

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